NACS 643 Homework #2
Spring 2009

For these problems, you will probably find the Matlab function "`randn()`" useful. It generates an array or matrix of random numbers with a Gaussian distribution (of mean 0 and standard deviation 1). You can then change the standard deviation by multiplying the given random number by the desired standard deviation (and, if necessary, you can change the mean by then adding the desired mean to that number). When generating such numbers, you may want to double check that the mean and standard deviation of the final random numbers are indeed what you intended them to be.

1) In this problem you will simulate the random-dot direction discrimination experiment. You will not need, however, to create the stimuli, or even the spike trains in response; you will only simulate the resultant spike rate as a (self-contained) random variable.
Denote the stimulus by plus or minus, corresponding to the two directions of motion. On each trial, choose the stimulus randomly with equal probability for the two cases (e.g. based on the sign of a Gaussian random number with zero mean). When the minus stimulus is chosen, generate the response rate of the neuron r as 20 Hz plus a random Gaussian term with a standard deviation of 10 Hz (set any rates that come out negative to zero). When the plus stimulus is chosen, generate the responses as $(20+10d)$ Hz plus a random Gaussian term with a standard deviation of 10 Hz, where d is the discriminability (again, set any rates that come out negative to zero). First, choose a threshold $z = 20 + 5d$, which is halfway between the means of the two response distributions. Whenever $r \geq z$, guess "plus", otherwise guess "minus".
   a) Over a large number of trials (1000, for example) determine how often you get the right answer for different d values. Plot the percent correct as a function of d over the range $0 \leq d \leq 10$.
   b) Next, by allowing z to vary over a range, plot ROC curves for several values of d (starting with $d = 2$). To do this, determine how frequently the guess is "plus" when the stimulus is, in fact, plus (this is β), and how often the guess is "plus" when the real stimulus is minus (this is α). Then, plot β versus α for z over the range $0 \leq z < 140$.

2) Simulate the responses of four interneurons in the cercal system of the cricket and check the accuracy of a vector decoding scheme. For a true wind direction θ, the average firing rates of the four interneurons should be generated as $\langle r_i \rangle = \left[50 \text{ Hz} \cos(\theta - \theta_i) \right]_+$ where $[\ ]_+$ indicates half-wave rectification (i.e. turn

negative values to zero), and $\theta_i = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$ for $i = 1, 2, 3, 4$. The actual rates, r_i , are then obtained by adding to these mean rates a random number chosen from a Gaussian distribution with zero mean and a standard deviation of 5 Hz (set any rates the come out negative to zero). From these rates, construct the x and y components of the population vector

$$x = \sum_{i=1}^{4} r_i \cos(\theta_i) \quad \text{and} \quad y = \sum_{i=1}^{4} r_i \sin(\theta_i)$$

and, from the direction of this vector, compute an estimate θ_{est} of the wind direction. Average the squared difference $(\theta - \theta_{\text{est}})^2$ over 1000 trials. The square root of this quantity is the error. Plot the error as a function of θ over the range $-90° \le \theta \le 90°$. Remember that Matlab requires that angles be in radians, and to convert angles to degrees for plotting requires multiplication by $360°/(2\pi)$.